# Constant stepsize gradient descent on smooth strongly convex function

Samuel Vaiter

Created: 2023-11-10.
Last update: 2023-11-10.
Status: inprogress.

# Strongly convex $L$-smooth function

## Proposition

*Let $f : \mathbb{R}^d \to \mathbb{R}$ be a $\mu$-strongly convex $L$-smooth function. For any $x^{(0)}$, the GD iterates $x^{(t)}$ with $t^{(t)} = \eta \in (0, \frac{2}{\mu+L}]$ converge towards the optimal point of $f$*

$$\|x^{(t)} - x^\star\|^2 \leq \left(1 - \frac{2\eta\mu L}{\mu + L}\right)^t \|x^{(0)} - x^\star\|^2.$$

*Moreover, if $\eta = \frac{2}{\mu+L}$, then (with $K_f = L/\mu$)*

$$\|x^{(t)} - x^\star\|^2 \leq \left(\frac{K_f - 1}{K_f + 1}\right)^{2t} \|x^{(0)} - x^\star\|^2$$

$$f(x^{(t)}) - f^\star \leq \frac{L}{2} \left(\frac{K_f - 1}{K_f + 1}\right)^{2t} \|x^{(0)} - x^\star\|^2.$$