Means of convergence

Samuel Vaiter

Created: 2023-11-08.
Last update: 2023-11-08.
Status: inprogress.

Mean of convergence

Convergence in norms of the iterates. Distance to a minimizer:

$$r^{(t)} = \|x^{(t)} - x^\star\| \leq \varepsilon.$$

Convergence to the set of minimizers. Distance to any minimizer:

$$r^{(t)} = d(x^{(t)}, \mathcal{S}) = \inf_{x^\star \in \mathcal{S}} \|x^{(t)} - x^\star\| \leq \varepsilon,$$

where $\mathcal{S}$ is the set of minimizer.

Convergence in value (0-order). Infimum value $f^* = \inf_{x \in \mathbf{dom}\, f} f(x)$

$$r^{(t)} = f(x^{(t)}) - f^* \leq \varepsilon,$$

First-order convergence. If f is differentiable,

$$r^{(t)} = \|\nabla f(x^{(t)})\| \leq \varepsilon.$$

Rate vs Complexity

- **Sublinear rates**

$$r^{(t)} \leq ct^{-\alpha}, \quad c > 0 \text{ and } \alpha > 0$$

  with complexity bound $(\frac{c}{\varepsilon})^{1/\alpha}$

- **Linear rates**

$$r^{(t)} \leq ce^{-qt}, \quad c > 0 \text{ and } q \in [0, 1)$$

  with complexity bound $\frac{1}{q}(\log c + \log \frac{1}{\varepsilon})$

- **Superlinear rates** (*e.g.*, quadratic)

$$r^{(t)} \leq c(r^{(t)})^2, \quad c > 0$$

  with complexity bound $\log c + \log\log \frac{1}{\varepsilon}$ (1 iteration = double precision)