

Local minima of convex functions are global

Samuel Vaiter

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$$\min_{x \in C} f(x), \quad (\text{Opt})$$

Theorem

Assume that $f : C \rightarrow \mathbb{R}$ is a convex function and C is a convex **closed** set. Then,

1. Any local solution of (Opt) is a global solution;
2. The set of global solutions of (Opt) is convex.

Proof idea

1. By contradiction.
2. Consider two solutions and study the segment joining them.