Descent direction

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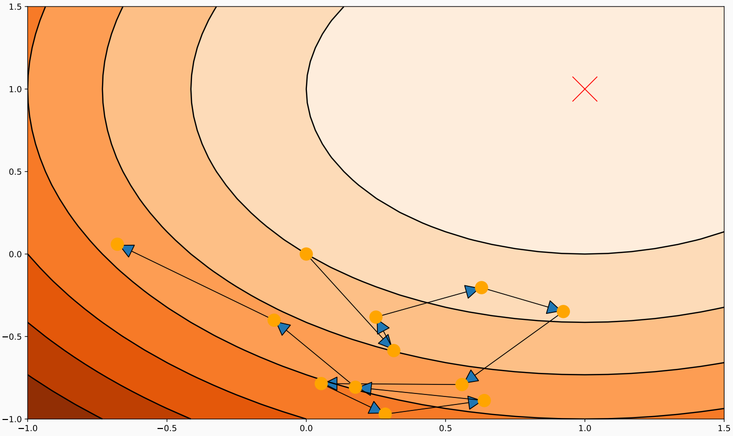
Descent direction

Iterative algorithm

$$x^{(t+1)} = x^{(t)} + \eta^{(t)} u^{(t+1)}$$



$$u^{(t+1)} \sim \mathcal{U}(-1, 1)$$
$$\eta^{(t)} = 0.6$$

Descent direction

Iterative algorithm

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Choice of direction? how to “optimaly choose” the direction $u^{(t+1)}$ such that $x^{(t+1)}$ is closer to a minimum of f?

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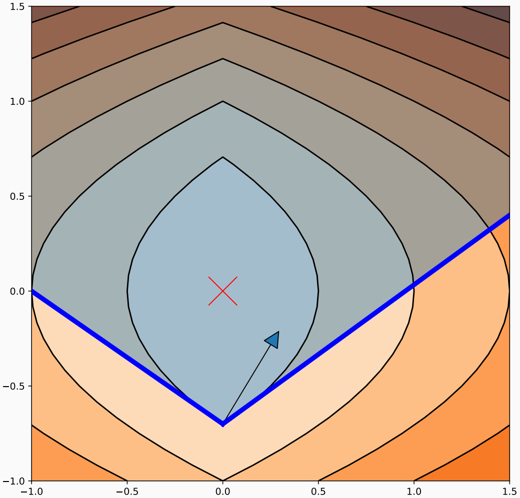
Definition

A *descent direction* $u \in \mathbb{R}^d$ of f at $x \in \mathbb{R}^d$ is vector such that

$$\exists \varepsilon > 0, \forall \eta \in (0, \varepsilon), \quad f(x + \eta u) < f(x).$$



Descent direction: differentiable case

Descent direction: $\exists \varepsilon > 0, \forall \eta \in (0, \varepsilon), \quad f(x + \eta u) < f(x)$

Characterization: $\langle \nabla f(x), u \rangle < 0$ (for f differentiable)

Why?

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Why?

Proposition (Antigradient is the steepest direction)

Let $f : \mathbb{R}^d \to \mathbb{R}$ be a differentiable function at $\bar{x} \in \mathbb{R}^d$. Then, the problem

$$\min_{\|u\|=1} \langle \nabla f(\bar{x}), u \rangle$$

has a unique solution $u^\star = -\frac{\nabla f(\bar{x})}{\|\nabla f(\bar{x})\|}$.

Proof idea: study $\phi_u(t) = f(\bar{x} + \eta u)$.