Convex set

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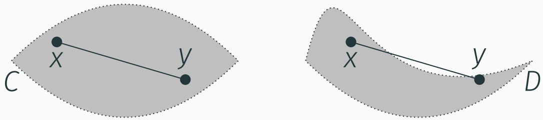
Convex set

A set C is convex if every segment of C is contained in C.



Definition (Convex set)

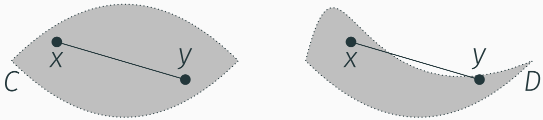
A set $C \subseteq \mathcal{X}$ is convex if

$$\forall (x, y) \in C^2, \forall \lambda \in [0, 1], \quad \lambda x + (1 - \lambda) y \in C$$

Exercice: Prove the equivalence between $[0, 1]$ and $(0, 1)$.

Convex set

A set C is convex if every open segment of C is contained in C.



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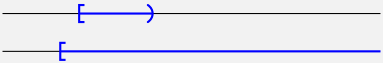
Examples of convex sets

Convex sets of $\mathbb{R}$ = intervals



Simplex & Orthant

nonnegative orthant

$$\mathbb{R}^d_{\geq 0} = \{x \ : \ x_i \geq 0\}$$

simplex

$$\Delta^{d-1} = \mathbb{R}^d_{\geq 0} \cap \{x \ : \ \sum_i x_i = 1\}$$

Affine hyperplanes & half-spaces

φ: linear functional over $\mathcal{X}$

hyperplane $H = \{x \in \mathcal{X} \ : \ \varphi(x) = \alpha\}$

half-space $H^- = \{x \in \mathcal{X} \ : \ \varphi(x) \leq \alpha\}$