

Complex step approximation

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Complex step approximation

$f : \mathbb{C} \rightarrow \mathbb{C}$ analytic function with $f(\mathbb{R}) \subseteq \mathbb{R}$ (loc. convergent power series $\sum_n a_n z^n$).

Taylor expansion at order 3

$$f(x + ih) = f(x) + ihf'(x) - \frac{h^2}{2}f''(x) + O(h^3).$$

Identifying, imaginary and real part, we have

$$\Re f(x + i\varepsilon) = f(x) + O(\varepsilon^2) \quad \text{and} \quad \Im f(x + i\varepsilon) = hf'(x) + O(\varepsilon^3).$$

$$f'(x) \approx \Im \frac{f(x + i\varepsilon)}{\varepsilon}$$

No round-off error!

